Conditional probability:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Bayes' Theorem:

Let A, B be events with P(B) ≠0. Then:

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

Or the extended alternative:

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B|A) * P(A) + P(B|\bar{A}) * P(\bar{A})}$$

Where Ā must be understand as not-A

Conditioning a random variable:

$$P(X = k \mid B) = \frac{P(X = k \cap B)}{P(B)}$$

Conditioning expectation:

$$E(X \mid B) = \sum_k k \cdot P(X = k \mid B)$$

Law of total probability:
If B1, B2, B3… is a partition of the sample space S, then for any event A we have:

$$P(A) = \sum_i P(A \cap B_i) = \sum_i P(A \mid B_i) P(B_i)$$

Law of total probability for expectations:

$$E(X) = E(X \mid A_1) P(A_1) + ... + E(X \mid A_n) P(A_n)$$

Normal distribution:

- **Standard:**

$$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$f_{aX+b}(x) = \frac{1}{a} f_X\left(\frac{1}{a}(x-b)\right)$$

- **General:**

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- **Standard deviation:**

$$\sigma = \sqrt{Var(X)} = \sqrt{\sigma^2}$$

- **Z value:**

$$Z = \frac{1}{\sigma}(X - \mu)$$

$$E(X) = \mu$$

$$Var(X) = \sigma^2$$

Several random variables:

- **The joint probability mass function of X,Y:**

$$p_{X,Y}(x,y) = P(X = x \cap Y = y)$$

$$0 \le p_{X,Y}(x,y) \le 1$$

$$\sum_x \sum_y p_{X,Y}(x,y) = 1$$

- **Marginal mass functions:**

$$p_X(x) = \sum_y p_{X,Y}(x,y)$$

$$p_Y(y) = \sum_x p_{X,Y}(x,y)$$

Given by rows and columns:

$$P(X = x \mid Y = y)$$

$$P(Y = y \mid X = x)$$

Continuous case - joint density function:

For all (x,y):

$$0 \le f_{X,Y}(x,y)$$

$$\int\int_S f_{X,Y}(x,y)\,dx\,dy = 1$$

- **Marginal densities:**

$$f_X(x) = \int f_{X,Y}(x,y)dy$$

$$f_Y(y) = \int f_{X,Y}(x,y)dx$$

- **Independence:**

If X,Y are independent:

$$P(X \in [r,s] \cap Y \in [u,v]) = P(X \in [r,s]) P(Y \in [u,v])$$

$$f_{X,Y}(x,y) = f_X(x) f_Y(y)$$

Other "independences" :

$$P(X = k \cap Y = l) = P(X = k) P(Y = l)$$

$$E(XY) = E(X)E(Y)$$

$$Var(X + Y) = Var(X) + Var(Y)$$

$$G_{X+Y}(s) = G_X(s) G_Y(s)$$

$$Cov(X,Y) = 0$$

Covariance:

$$Cov(X,Y) = E((X-\mu_X)(Y-\mu_Y))$$
$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

Properties:

$$Cov(X,Y) = Cov(Y,X)$$
$$Cov(X,X) = Var(X)$$
$$Cov(aX+bY,Z) = aCov(X,Z) + bCov(Y,Z)$$

Variance and covariance:

$$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$$

Correlation:

$$(Cov(X,Y))^2 \le Var(X)Var(Y)$$

$$Cor(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}}$$

$$-1 \le Cor(X,Y) \le 1$$

Properties:

$$Cov(aX+b, cX+d) = acVar(X)$$

$$Cor(aX+b, cX+d) = \frac{ac}{|ac|} = \pm 1$$

Distributions - overview:

- **Bernoulli (Yes/No questions)**

If a coin is tossed once, what is the probability that it comes up heads?

- **Binomial**

- The number of trials is fixed.
- The number of successes is a random variable.

If a coin is tossed 20 times, what is the probability heads come up exactly 14 times?

What is the probability of getting more than half the answers right in a test of 10 questions if for each question independently I am 60% likely to answer it correctly.

- **Geometric**

If a coin is repeatedly tossed, what is the probability the *first* time heads appears occurs on the 8th toss?

Roll two D6 repeatedly. How likely is it that I first have a roll where the product of the 2 numbers is 12 on my 11th roll?

- **Negative binomial**

- The number of successes is fixed.
- The number of trials is a random variable.

If a coin is repeatedly tossed, what is the probability the third time heads appear occurs on the 9th toss?

- **Poisson**

- Closely approximates the binomial distribution if n is large and p is small.

What is the probability there will be 4 car accidents on a university campus in a given week?

Customers arrive at a bank machine at an average rate of 2 per minute. How likely is it that 5 or more arrive in a single given minute?

- **Exponential**

I am waiting for the next lightening flash in a storm. At any time, the probability of the next flash being in the next ΔtΔt seconds is 0.05Δt0.05Δt (in the limit as Δt→0Δt→0). How likely is it that the flash will occur in the next 4 seconds?